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OPTIMUM THICKNESS OF A COOLED WALL IN LOCAL PULSE-PERIODIC HEATING

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Using the methods of mathematical simulation. We investigated the characteristic features of formation of a stationary temperature field in a cooled plane isotropic wall subjected to local pulse-periodic heating. The possibility of the existence of an optimum wall thickness that ensures a minimum stationary temperature of its most heated point has been established.

For practical applications of the mathematical theory of heat conduction [1–3], of special interest are the problems of controlling the temperature state of a structure [4–11], optimization, and estimation of the effective values of thermophysical and geometric parameters of their individual elements [12–15]. A correct choice of the optimum parameters depends to a great extent on the space-time structure of the regime of thermal action realized.

Great attention in theoretical investigations is paid to spatial distributions (with the Gaussian-type intensity) and concentrated (of circular and annular shapes, and of other right geometrical shapes) heat fluxes. Thus, in particular, for a heat flux with a Gaussian-type intensity the problem of determination of the "optimal thickness of the wall subjected local heating" has been solved [16] and also sufficient conditions of the existence of the optimum thickness of a cooled coated wall being pulse-periodically heated have been determined [17]. As far as the present authors are aware, for concentrated heat fluxes similar results are absent, which can be explained by the specific properties of the process of formation of corresponding temperature fields [18, 19].

In the present work, we consider a plane isotropic wall of constant thickness h, one of the surfaces of which is exposed to the influence of both the surrounding medium with a constant temperature T_c^0 and heat-transfer coefficient α^0 and a concentrated axisymmetrical heat flux $q(r)$ in a pulse-periodic regime and the other surface of which is cooled by the external medium with a constant temperature T_c^h and heat-transfer coefficient α^h. The main aim of the investigations carried out is analysis of the characteristic features of a stationary temperature field in the cooled wall on its exposure to a concentrated axisymmetric heat flux.

To attain the aim set, we used a nonstationary mathematical model of the process of formation of a stationary temperature field in the wall. This allowed us to take into account the influence of the nonstationary nature of the acting heat-flux regime on the temperature field of the wall cooled and, by using the limiting transition (with unlimited increase in the time variable), to find the corresponding stationary temperature field that is the object of a subsequent parametric analysis.

In conformity with the aim set and initial assumptions, we will avail ourselves of the following mathematical model:

$$\frac{\partial \theta}{\partial \mathrm{Fo}} = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial \theta}{\partial \rho}\right) + \frac{\partial^2 \theta}{\partial x^2}, \quad \rho \geq 0, \quad 0 < x < H, \quad \mathrm{Fo} > 0; \tag{1}$$

$$\theta(\rho, x, \mathrm{Fo})\big|_{\mathrm{Fo}=0} = 0; \tag{2}$$

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$$\left.\frac{\partial\theta(\rho, x, \mathrm{Fo})}{\partial x}\right|_{x=0} = \mu\mathrm{Bi}\,(\theta(\rho, x, \mathrm{Fo})|_{x=0} - 1) - Q(\rho)\,\varphi(\mathrm{Fo})\,; \tag{3}$$

$$\left.\frac{\partial\theta(\rho, x, \mathrm{Fo})}{\partial x}\right|_{x=H} = \mathrm{Bi}\,(\theta_{\tilde{n}} - \theta(\rho, x, \mathrm{Fo})|_{x=H})\,, \tag{4}$$

where at any fixed $\mathrm{Fo} > 0$ and $x \in (0; H)$ the quantities $\theta(\rho, x, \mathrm{Fo})$ and $Q(\rho)$ as functions of ρ are the inverted transforms of the Hankel integral zero-order transformation [2];

$$\varphi(\mathrm{Fo}) = \sum_{n=0}^{\infty}\left\{\eta\left[\mathrm{Fo} - n(\mathrm{Fo}^* + \Delta\mathrm{Fo}^*)\right] - \eta\left[\mathrm{Fo} - \mathrm{Fo}^* - n(\mathrm{Fo}^* + \Delta\mathrm{Fo}^*)\right]\right\};$$

$$x = \frac{z}{r_1}\,;\quad \rho = \frac{r}{r_1}\,;\quad \mathrm{Fo} = \frac{at}{r_1^2}\,;\quad \theta = \frac{T - T_0}{T_{\tilde{n}}^0 - T_0}\,;\quad \theta_{\tilde{n}} = \frac{T_{\tilde{n}}^h - T_0}{T_{\tilde{n}}^0 - T_0}\,;$$

$$\mathrm{Bi} = \frac{\alpha^h r_1}{\lambda}\,;\quad R = \frac{r_2}{r_1}\,;\quad H = \frac{h}{r_1}\,;\quad \mu = \frac{\alpha^0}{\alpha^h}\,;\quad Q = \frac{q r_1}{\lambda(T_{\tilde{n}}^0 - T_0)}\,.$$

The form of the function $Q(\rho)$ depends on the implemented regime of exposure to a concentrated heat flux. For an annular heat flux with the inner radius $R \in (0, 1)$ of the heated zone

$$Q(\rho) = Q_0\,[\eta(\rho - R) - \eta(\rho - 1)]\,.$$

The regime of the effect of the circular heat flux can be considered as a particular case of the annular one with $R \to +0$.

By virtue of the linearity of the initial mathematical model (1)–(4), the function $\theta(\rho, x, \mathrm{Fo})$ that describes evolution of the temperature field of interest can be represented as a sum of its three components:

$$\theta(\rho, x, \mathrm{Fo}) = \theta_1(x, \mathrm{Fo}) + \theta_2(\rho, x, \mathrm{Fo}) + \theta_3(\rho, x, \mathrm{Fo})\,.$$

The function $\theta_1(x, \mathrm{Fo})$ satisfies (1)–(4) at $Q_0 = 0$, i.e., it is the solution of the problem on temperature distribution in a plane isotropic wall with a contact initial temperature T_0 on the condition that heat exchange with the surrounding media with constant temperatures T_c^0 and T_c^h, respectively, is implemented by the Newton law [2].

The function $\theta_2(\rho, x, \mathrm{Fo})$ satisfies Eq. (1), initial condition (2), boundary condition (3) at $\mu = 0$, and a homogeneous analog of boundary condition (4):

$$\left.\frac{\partial\theta_2(\rho, x, \mathrm{Fo})}{\partial x}\right|_{x=H} = -\,\mathrm{Bi}\theta_2(\rho, x, \mathrm{Fo})|_{x=H}\,, \tag{5}$$

i.e., it determines the process of formation of a temperature field in a plane isotropic wall, one of the surfaces of which is cooled by the external medium of constant temperature equal to the initial temperature of the wall and the other of which is subjected to local concentrated heat flux in a pulse-periodic regime.

Interpretation of the function $\theta_3(\rho, x, \mathrm{Fo})$ that satisfies Eq. (1), initial condition (2), and the boundary conditions

$$\left.\frac{\partial\theta_3(\rho, x, \mathrm{Fo})}{\partial x}\right|_{x=0} = \mu\mathrm{Bi}\,(\theta_2(\rho, x, \mathrm{Fo})|_{x=0} + \theta_3(\rho, x, \mathrm{Fo})|_{x=0})\,;$$

$$\left.\frac{\partial\theta_3(\rho, x, \mathrm{Fo})}{\partial x}\right|_{x=H} = -\mathrm{Bi}\theta_3(\rho, x, \mathrm{Fo})\big|_{x=H},$$

is similar to the interpretation of the function $\theta_1(x, \mathrm{Fo})$.

With account for the analysis carried out and the well-known results of [17], we may state that the attainment of the goal set is directly associated with studying the properties of the function $\theta_2(\rho, x, \mathrm{Fo})$ at $\mathrm{Fo} = +\infty$ and $x = 0$. Let further

$$U(\rho, x, s) = \int_0^{+\infty} \exp(-s\mathrm{Fo})\, \theta_2(\rho, x, \mathrm{Fo})\, d\mathrm{Fo} \tag{6}$$

be the transform of the integral Laplace transformation [2] of the solution $\theta_2(\rho, x, \mathrm{Fo})$ of problem (1)–(3), (5) at $\mu = 0$, and

$$V(p, x, s) = \int_0^{+\infty} U(\rho, x, s)\, \rho J_0(p\rho)\, d\rho, \quad G(p) = \int_0^{+\infty} Q(\rho)\, \rho J_0(p\rho)\, d\rho \tag{7}$$

be the transforms of the integral Hankel zero-order transformation [2] of the functions $U(p, x, s)$ and $Q(\rho)$, respectively. Using the well-known results of the theory of integral transformations [2], we may show that the transform $V(p, x, s)$ is the solution of the following boundary-value problem:

$$\frac{d^2V}{dx^2} = (s + p^2)\, V, \quad 0 < x < H; \tag{8}$$

$$\left.\frac{dV(p, x, s)}{dx}\right|_{x=0} = -G(p)\, \frac{1}{s}\, \frac{1 - \exp(-s\mathrm{Fo}^*)}{1 - \exp[-s(\mathrm{Fo}^* + \Delta\mathrm{Fo}^*)]}; \tag{9}$$

$$\left.\frac{dV(p, x, s)}{dx}\right|_{x=H} = -\mathrm{Bi}V(p, x, s)\big|_{x=H}, \tag{10}$$

where for the annular heat flux the transform (7) of the inverted transform $Q(\rho)$ is defined by the equality [20, 21]

$$G(p) = [J_1(p) - RJ_1(Rp)]/p.$$

The solution of Eq. (8) can be presented in the form [17]

$$V(p, x, s) = c_1(p, s) \exp(-x\sqrt{s + p^2}) + c_2(p, s) \exp(x\sqrt{s + p^2})$$

and it must satisfy boundary conditions (9) and (10), which leads to a system of algebraic equations for finding the functionals $c_i(p, s)$ and $i \in |1, 2|$:

$$\sqrt{s + p^2}\, [c_2(p, s) - c_1(p, s)] = -G(p)\, \frac{1}{s}\, \frac{1 - \exp(-s\mathrm{Fo}^*)}{1 - \exp[-s(\mathrm{Fo}^* + \Delta\mathrm{Fo}^*)]}; \tag{11}$$

$$\sqrt{s + p^2}\, [c_2(p, s) \exp(H\sqrt{s + p^2}) - c_1(p, s) \exp(-H\sqrt{s + p^2})] =$$

$$= -\,\mathrm{Bi}\,[c_1(p,s)\exp(-H\sqrt{s+p^2}) + c_2(p,s)\exp(H\sqrt{s+p^2})]\,. \tag{12}$$

It is the determination of the functionals $c_i(p, s)$, $i \in |1, 2|$, satisfying the system (11)–(12) that completes the solution of the problem of finding the temperature field of the cooled wall subjected to local heating in transforms of the used integral Laplace (6) and Hankel (7) transformations.

To attain the aim set, it is sufficient to know the developed temperature $\theta_2(\rho, x, +\infty)$ on the surface of the plane isotropic wall. Then, in the transforms of the integral Laplace and Hankel transforms at $x = 0$ the solution of problem (8)–(10) can be presented in the following form:

$$V(p,0,s) = \frac{G(p)}{s\sqrt{s+p^2}}\,\frac{1-\exp(-s\mathrm{Fo}^*)}{1-\exp[-s(\mathrm{Fo}^*+\Delta\mathrm{Fo}^*)]}\,\frac{1-\exp(-2H\sqrt{s+p^2})\dfrac{\mathrm{Bi}-\sqrt{s+p^2}}{\mathrm{Bi}+\sqrt{s+p^2}}}{1+\exp(-2H\sqrt{s+p^2})\dfrac{\mathrm{Bi}-\sqrt{s+p^2}}{\mathrm{Bi}+\sqrt{s+p^2}}}\,.$$

Having inverted the integral Hankel zero-order transformation [2] and used the limiting theorem of operational calculus [2], we find the stationary distribution of temperature on the surface of a plane isotropic wall in a pulse-periodic regime of the action of an annular heat flux:

$$\theta_2(\rho,0,+\infty) = \lim_{s\to 0} s\int_0^{+\infty} V(p,0,s)\,pJ_0(p\rho)\,dp = Q_0\,\frac{\mathrm{Fo}^*}{\mathrm{Fo}^*+\Delta\mathrm{Fo}^*}\times$$

$$\times\int_0^{+\infty}\frac{J_1(p)-RJ_1(Rp)}{p}\,J_0(p\rho)\,\frac{1-\exp(-2Hp)\dfrac{\mathrm{Bi}-p}{\mathrm{Bi}+p}}{1+\exp(-2Hp)\dfrac{\mathrm{Bi}-p}{\mathrm{Bi}+p}}\,dp\,,\quad \rho\ge 0\,. \tag{13}$$

At the limiting values of the wall thicknesses $H = +0$ and $H = +\infty$, the integral on the right-hand side of equality (13) can be calculated explicitly [21–23]:

$$\theta_2(\rho,0,+\infty)\big|_{H=+0} = \frac{Q_0}{\mathrm{Bi}}\,\frac{\mathrm{Fo}^*}{\mathrm{Fo}^*+\Delta\mathrm{Fo}^*}\int_0^{+\infty}\{J_1(p)-RJ_1(Rp)\}\,J_0(p\rho)\,dp =$$

$$= \frac{Q_0}{\mathrm{Bi}}\,\frac{\mathrm{Fo}^*}{\mathrm{Fo}^*+\Delta\mathrm{Fo}^*}\begin{cases}0\,, & 0\le\rho<R\,;\\ 1\,, & R<\rho<1\,;\\ 0\,, & \rho>1\,;\end{cases}$$

$$\theta_2(\rho,0,+\infty)\big|_{H=+\infty} = \frac{Q_0\mathrm{Fo}^*}{\mathrm{Fo}^*+\Delta\mathrm{Fo}^*}\int_0^{+\infty}p^{-1}\{J_1(p)-RJ_1(Rp)\}\,J_0(p\rho)\,dp\,. \tag{14}$$

Depending on the value of ρ, the integral on the right-hand side of equality (14) can be represented differently [20, 21, 23] as

$$\theta_2(\rho,0,+\infty)\big|_{H=+\infty} = \frac{Q_0\mathrm{Fo}^*}{\mathrm{Fo}^*+\Delta\mathrm{Fo}^*}\,\frac{2}{\pi}\left[\mathrm{E}(\rho) - R\mathrm{E}\left(\frac{\rho}{R}\right)\right],\quad 0\le\rho<R\,;$$

$$\theta_2(\rho,0,+\infty)\big|_{H=+\infty} = \frac{Q_0\mathrm{Fo}^*}{\mathrm{Fo}^*+\Delta\mathrm{Fo}^*}\,\frac{2}{\pi}\left[\mathrm{E}(\rho) - \rho\left\{\mathrm{E}\left(\frac{R}{\rho}\right) - \left(1-\frac{R^2}{\rho^2}\right)\mathrm{K}\left(\frac{R}{\rho}\right)\right\}\right],\quad R<\rho<1\,;$$

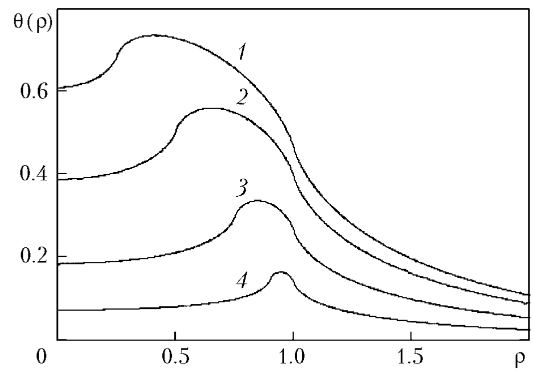

Fig. 1. Stationary temperature profile of the surface $x = 0$ of the cooled wall at different values of the parameter R $\in$ (0, 1) in a pulse-periodic regime of exposure to an annular heat flux: 1) R = 0.25; 2) 0.5; 3) 0.75; 4) 0.9.

$$\theta_2(\rho, 0, +\infty)\big|_{H=+\infty} = \frac{Q_0\mathrm{Fo}^*}{\mathrm{Fo}^* + \Delta\mathrm{Fo}^*}\frac{2}{\pi}\rho\left[\mathrm{E}\left(\frac{1}{\rho}\right) - \mathrm{E}\left(\frac{R}{\rho}\right) - \left(1 - \frac{1}{\rho^2}\right)\mathrm{K}\left(\frac{1}{\rho}\right) + \left(1 - \frac{R^2}{\rho^2}\right)\mathrm{K}\left(\frac{R}{\rho}\right)\right], \quad \rho > 1 .$$

Here, in conformity with the properties of full elliptical integrals [21–23],

$$\theta_2(0, 0, +\infty)\big|_{H=+\infty} = \frac{Q_0\mathrm{Fo}^*}{\mathrm{Fo}^* + \Delta\mathrm{Fo}^*}(1 - R) ,$$

$$\theta_2(R, 0, +\infty)\big|_{H=+\infty} = \frac{2}{\pi}\frac{Q_0\mathrm{Fo}^*}{\mathrm{Fo}^* + \Delta\mathrm{Fo}^*}[\mathrm{E}(R) - R] ,$$

$$\theta_2(1, 0, +\infty)\big|_{H=+\infty} = \frac{2}{\pi}\frac{Q_0\mathrm{Fo}^*}{\mathrm{Fo}^* + \Delta\mathrm{Fo}^*}[1 - \mathrm{E}(R) + (1 - R^2)\,\mathrm{K}(R)] .$$

Since $0 < R < 1$, we may note that the inequality

$$\theta_2(1, 0, +\infty)\big|_{H=+\infty} < \theta_2(R, 0, +\infty)\big|_{H=+\infty}$$

is satisfied at any $R \in (0, 1)$.

The results of calculations of the stationary temperature profile of the wall surface subjected to the pulse-periodic effect of an annular heat flux at different values of the parameter R are partially presented in Fig. 1. The calculations were carried out using equality (13) at H = 1 and Bi = 1. For convenience of representation of the graphical information, here and in what follows (see Fig. 2) the quantity $\theta(\rho) = ([\mathrm{Fo}^* + \Delta\mathrm{Fo}^*]/Q_0\mathrm{Fo}^*)\theta_2(\rho, 0, +\infty)$ — a value proportional to the developed temperature of the heated wall surface — is laid down on the vertical axis.

Passing in equality (13) to the limit for $R \to +0$, we obtain the stationary distribution of temperature on the surface of a plane isotropic wall exposed to the influence of a circular heat flux in a pulse-periodic regime:

$$\theta_2(\rho, 0, +\infty)\big|_{R=+\infty} = Q_0\frac{\mathrm{Fo}^*}{\mathrm{Fo}^* + \Delta\mathrm{Fo}^*}\int_0^{+\infty}\frac{J_1(p)}{p}J_0(p\rho)\frac{1 - \exp(-2Hp)\dfrac{\mathrm{Bi} - p}{\mathrm{Bi} + p}}{1 + \exp(-2Hp)\dfrac{\mathrm{Bi} - p}{\mathrm{Bi} + p}}dp ; \quad \rho \ge 0 . \tag{15}$$

Similarly, we may obtain the values of temperature for a circular heat flux at the limiting values of the wall thickness H = +0 and H = +∞ [21–23]:

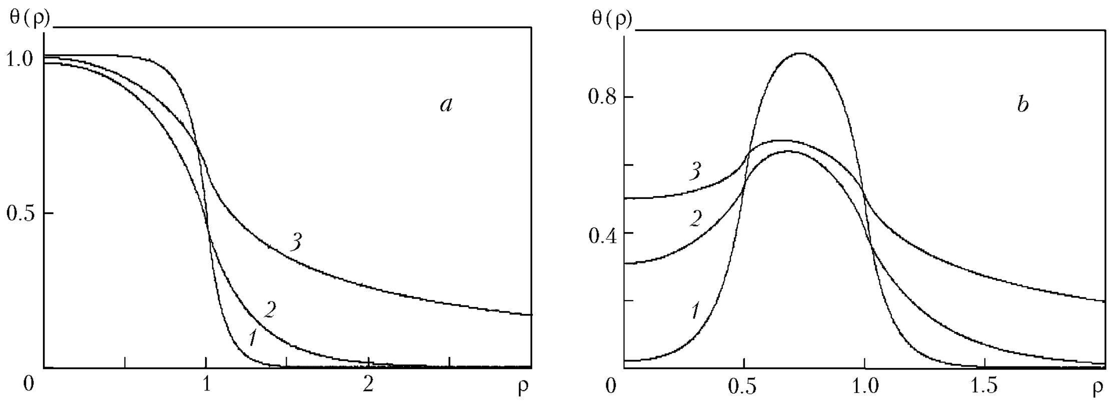

Fig. 2. Stationary temperature profile of the surface $x = 0$ of the cooled wall of different thicknesses H in a pulse-periodic regime of exposure to a circular (a) and annular (b) heat fluxes 1) $H = 0.01$; 2) $H = 0.1$; 3) $H = +\infty$.

$$\theta_2(\rho, 0, +\infty)\Big|_{\substack{H=+0\\R=+0}} = \frac{Q_0}{\mathrm{Bi}}\frac{\mathrm{Fo}^*}{\mathrm{Fo}^* + \Delta\mathrm{Fo}^*}\int_0^{+\infty} J_1(p)\, J_0(p\rho)\, dp = \frac{Q_0}{\mathrm{Bi}}\frac{\mathrm{Fo}^*}{\mathrm{Fo}^* + \Delta\mathrm{Fo}^*}\,\eta(1-\rho) =$$

$$= \frac{Q_0}{\mathrm{Bi}}\frac{\mathrm{Fo}^*}{\mathrm{Fo}^* + \Delta\mathrm{Fo}^*}\begin{cases}1, & 0 \le \rho < 1;\\ 0, & \rho > 1;\end{cases}$$

$$\theta_2(\rho, 0, +\infty)\Big|_{\substack{H=+\infty\\R=+0}} = \frac{Q_0\mathrm{Fo}^*}{\mathrm{Fo}^* + \Delta\mathrm{Fo}^*}\int_0^{+\infty} p^{-1} J_1(p)\, J_0(p\rho)\, dp = \frac{2}{\pi}\frac{Q_0\mathrm{Fo}^*}{\mathrm{Fo}^* + \Delta\mathrm{Fo}^*}\times$$

$$\times\left\{\mathrm{E}(\rho)\,\eta(1-\rho) + \rho\left[\mathrm{E}\left(\frac{1}{\rho}\right) - \left(1 - \frac{1}{\rho^2}\right)\mathrm{K}\left(\frac{1}{\rho}\right)\right]\eta(\rho - 1)\right\} =$$

$$= \frac{2Q_0}{\pi}\frac{\mathrm{Fo}^*}{\mathrm{Fo}^* + \Delta\mathrm{Fo}^*}\begin{cases}\mathrm{E}(\rho), & 0 \le \rho < 1;\\ \rho\left[\mathrm{E}\left(\frac{1}{\rho}\right) - \left(1 - \frac{1}{\rho^2}\right)\mathrm{K}\left(\frac{1}{\rho}\right)\right], & \rho > 1.\end{cases}$$

Here

$$\theta_2(0, 0, +\infty)\Big|_{\substack{H=+\infty\\R=+0}} = \frac{Q_0\mathrm{Fo}^*}{\mathrm{Fo}^* + \Delta\mathrm{Fo}^*}; \quad \theta_2(1, 0, +\infty)\Big|_{\substack{H=+\infty\\R=+0}} = \frac{2}{\pi}\frac{Q_0\mathrm{Fo}^*}{\mathrm{Fo}^* + \Delta\mathrm{Fo}^*}.$$

The results of calculations of the stationary temperature profile of the surface $x = 0$ of the wall exposed to the pulse-periodic effect of a circular ($R = +0$) heat flux and an annular ($R = 0.5$) one, respectively, are presented in Fig. 2a and b. The calculation was performed at Bi = 1 and at different values of the wall thickness H. The fundamental difference between the shapes of the stationary temperature profile during the action of the circular and annular heat fluxes should be noted. This, in particular, leads to the dependence of the point of the temperature maximum of the surface $x = 0$ of the wall on the realized regime of thermal effect: in the first case it is located on the symmetry axis of the wall at $\rho = 0$, whereas in the second case it lies on the circle $\rho = \rho^*$, where $\rho^* \in (R, 1)$, with the value of the radius ρ^* of the circle being decisively dependent on the parameter $R \in (0, 1)$ and also on the wall thickness and heat-transfer intensity on its surface $x = H$.

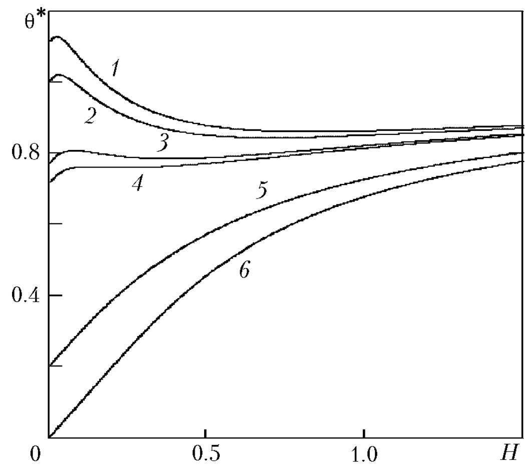

Fig. 3. Dependence of the dimensionless quantity θ^* on the thickness H of the wall at different values of the Biot number in a pulse-periodic regime of exposure to a circular heat flux: 1) Bi = 0.9; 2) 1; 3) 1.3; 4) 1.4; 5) 5; 6) Bi = $+\infty$.

The result obtained is of fundamental importance in determining the conditions of the existence and determination of the optimum thickness of a cooled wall if the condition of provision of the minimum developed temperature at the most heated point of the wall was used as the optimality criterion [16, 17]. This, in particular, makes it possible to determine sufficient conditions for the existence of the optimum thickness of a one-layer wall [16] and of a coated wall [17] exposed to an axisymmetric heat flux having the Gaussian-type intensity.

We will consider in more detail the problem of finding the most heated point of the cooled wall exposed to concentrated heat fluxes. According to equality (15) and the properties of the Bessel function $J_0(y)$ that attains a maximum at $y = 0$ [23], for a circular heat flux the point (0, 0) of the wall turns out to be the most heated one. Its steady-state temperature is defined as

$$\theta_2(0, 0, +\infty)\big|_{R=+\infty} = Q_0 \frac{\mathrm{Fo}^*}{\mathrm{Fo}^* + \Delta\mathrm{Fo}^*} \int_0^{+\infty} \frac{J_1(p)}{p} \frac{1 - \exp(-2Hp)\dfrac{\mathrm{Bi} - p}{\mathrm{Bi} + p}}{1 + \exp(-2Hp)\dfrac{\mathrm{Bi} - p}{\mathrm{Bi} + p}}\, dp\,, \quad \rho \ge 0\,. \tag{16}$$

We will limit the determination of the radius $\rho^* \in (R, 1)$ of the circle with points of the temperature maximum of the $x = 0$ surface of the wall exposed to the annular heat flux to the simplest case $H = +\infty$. Having differentiated equality (14) with respect to the variable ρ, we obtain

$$\left.\frac{\partial\theta_2(\rho, 0, +\infty)}{\partial\rho}\right|_{H=+\infty} = -\frac{Q_0\mathrm{Fo}^*}{\mathrm{Fo}^* + \Delta\mathrm{Fo}^*} \int_0^{+\infty} \left\{J_1(p) - RJ_1(Rp)\right\} J_1(p\rho)\, dp =$$

$$= -\frac{Q_0\mathrm{Fo}^*}{\mathrm{Fo}^* + \Delta\mathrm{Fo}^*} \left(\int_0^{+\infty} J_1(y)\, J_1(\rho y)\, dy - \int_0^{+\infty} J_1(y)\, J_1\left(\frac{\rho}{R} y\right) dy\right). \tag{17}$$

Using equality (17), we can easily see that

$$\exists \lim_{\rho \to 0} \left.\frac{\partial\theta_2(\rho, 0, +\infty)}{\partial\rho}\right|_{H=+\infty} = 0\,,$$

and we may state (see Figs. 1 and 2b) that (0, 0) is the point of the local minimum. Having calculated the integrals on the right-hand side of equality (17) [20], we obtain an equation for finding ρ^*:

$$\sum_{k=0}^{+\infty} \frac{(2k+1)\,!!\,(2k-1)\,!!}{2^{2k+1}\,(k+1)\,!k!} \left\{\rho^{2k+1} - \frac{R^{2k+2}}{\rho^{2k+2}}\right\} = 0\ ,$$

from which it follows that $\rho^* \approx \sqrt{R}$. In particular, at $H = +\infty$ and $R = 0.5$, using numerical methods, we may determine the value of ρ at which the temperature $\theta_2(\rho, 0, +\infty)$ attains its maximum value, i.e., find the most heated point on the wall. In the indicated particular case, $\rho^* = 0.6601058$ and $\sqrt{R} = 0.7071067$.

Figure 3 presents some of the results of calculations of the dependence of the dimensionless temperature $\theta_2(0, 0, +\infty)\big|_{R=+0}$ of the most heated point of the wall exposed to a pulse-periodic circular heat flux on its thickness H at different values of the Biot number. For the convenience of representation of graphical information, here the wall thickness H is plotted along the horizontal axis and the quantity $\theta^* = ([\mathrm{Fo}^* + \Delta\mathrm{Fo}^*]/Q_0\mathrm{Fo}^*)\theta_2(0, 0, +\infty)\big|_{R=+0}$, which is the quantity proportional to the steady-state temperature of the most heated point of the considered wall, is plotted along the vertical axis.

The results of the computational experiments carried out with the use of equality (16) and partially presented in Fig. 3 point to the possibility of existence of the optimum wall thickness that ensures a minimum steady-state temperature of its most heated point on exposure to a pulse-periodic circular heat flux. An increase in the intensity of heat transfer on the surface $x = H$ of the wall (increase in Bi) leads to a monotonic decrease (to zero) in the optimum wall thickness. We also note that in contrast to the pulse-periodic regime of the effect of a Gaussian-type axisymmetric heat flux [17], in the considered regimes of the influence of concentrated axisymmetric heat fluxes there is a nonzero wall thickness that ensures not only a minimum but also a maximum steady-state temperature of its most heated point.

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NOTATION

a, thermal diffusivity, m^2/sec; Bi, Biot number; E(·), full elliptical integral of the second kind; Fo, Fourier number; Fo* and ΔFo*, durations of active and passive phases of the action of pulse-periodic heat flux; h, wall thickness, m; H, dimensionless wall thickness; $J_\nu(\cdot)$, Bessel function of the first kind of order ν; K(·), full elliptical integral of the first kind; p, parameter of the Hankel integral transformation; q, density (intensity) of a heat flux, W/m^2; Q, dimensionless heat-flux density; r, spatial variable, m; r_2 and r_1, inner and outer radii of the zone of effect of an annular heat flux, m; $R \in (0, 1)$, dimensionless inner radius of the zone of heating by an annular heat flux; s, parameter of the Laplace integral transformation; T, temperature, K; t, time, sec; x, dimensionless spatial variable; z, spatial variable, m; α, heat-transfer coefficient, W/(m^2·K); $\eta(\cdot)$, Heaviside function; θ, dimensionless temperature; λ, thermal conductivity, W/(m·K); μ, similarity simplex as a measure of the relationship between the coefficients of heat transfer on the surfaces $x = 0$ and $x = h$ of the wall; ρ, dimensionless spatial variable. Superscripts: 0 and h denote the surfaces $x = 0$ and $x = h$ of the wall, respectively. Subscripts: c, external medium; 0, initial state of the wall.

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